Homework 2: Line Search

Name:

Instructions: For problems 1,3, and 4 submit a PDF containing plots with the initial condition chosen as $(0.xy, 0.z)$ where xyz are the three digits in your UKID. For these plots, choose tolerance as $1e^{-4}$ and max iterations sufficiently large enough for the tolerance condition to be met. For problem 2, submit a scalar value mentioned in the same PDF.

Problem 1: *(25 points)* Write a program to solve the following unconstrained minimization problem:

$$\min_x f(x) = (1 - x_1)^2 + 10(x_2 - x_1^2)^2$$

using the steepest descent algorithm

$$x_{k+1} = x_k - \alpha \nabla f_k,$$

with **constant $\boldsymbol{\alpha}$**. Plot:
a) the iterates x_k as points in the plane $\mathbb{R}^2$.
b) the $\log(f(x_k))$ (base e) against iteration number k.

Your code should accept the following variables:

- `start_point`: Initial point in $\mathbb{R}^2$
- `alpha`: step size variable α
- `max_iters` : maximum number of iterations
- `tol` : tolerance value for stopping condition based on ∇f_k

Problem 2: *(25 points)* Write a program that identifies the largest value of α – accurate up to three significant figures – for which the algorithm converges when starting from $x_0 = (0, 1)$.

Hint: convergence here is defined by $\| \nabla f \| <$ `tol`. If the number of iterations are too small, the convergence criterion may not be evaluated correctly.

Problem 3: *(25 points)* Write a program to solve the following unconstrained minimization problem:

$$\min_x f(x) = (1 - x_1)^2 + 10(x_2 - x_1^2)^2$$

using the steepest descent algorithm

$$x_{k+1} = x_k - \alpha_k \nabla f_k,$$

with α_k chosen using **back-tracking line search** with your choice of parameters. Plot:

a) the iterates x_k as points in the plane $\mathbb{R}^2$.
b) the $\log(f(x_k))$ (base e) against iteration number k.

Your code should accept the following variables:

- `start_point`: Initial point in $\mathbb{R}^2$
- `max_iters` : maximum number of iterations
- `tol` : tolerance value for stopping condition based on ∇f_k

Problem 4: *(25 points)* Write a program to solve the following unconstrained minimization problem:

$$\min_x f(x) = (1 - x_1)^2 + 10(x_2 - x_1^2)^2$$

using **Newton's method**

$$x_{k+1} = x_k - (\nabla^2 f_k)^{-1} \nabla f_k.$$

Plot:
a) the iterates x_k as points in the plane $\mathbb{R}^2$.
b) the $\log(f(x_k))$ (base e) against iteration number k.

Your code should accept the following variables:

- `start_point`: Initial point in $\mathbb{R}^2$
- `max_iters` : maximum number of iterations
- `tol` : tolerance value for stopping condition based on ∇f_k