ME 599/699 Robot Modeling & Control
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Independent Joint Control

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Introduction

- Derived an EL model for a torque-driven robot
- Torques from voltage-controlled PMDC motor $\implies$ individual motor model
- Combine motor and link models for link angle dynamics
- Extend model by including flexibility

Proportional Control

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- Proportional Control: $u(t) = -k_p(\theta_m(t) - \theta_d(t))$
- Closed-loop model:

$$(Js^2 + Bs)\Theta_m(s) = -k_p\Theta_m(s) + k_p\Theta_d(s) - D(s), \tag{2}$$

or

$$\Theta_m(s) = \frac{k_p}{Js^2 + Bs + k_p}\Theta_d(s) - \frac{1}{Js^2 + Bs + k_p}D(s). \tag{3}$$

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  - In practice, it's a bad idea due to sensor noise and actuator nonlinearities

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- As long as $k_p > 0$, $k_d > 0$, and disturbances are bounded, the closed loop system is stable.
- Similar behavior as P control for set-point regulation, with some more control on transient behavior
- In practice, full control on transients limited by actuator saturation

Proportional-Integral-Derivative Control

- Add an integral term:

$$u(t) = -k_p(\theta_m(t) - \theta_d(t)) - K_d\dot{\theta}_m(t) - k_I \int_0^t (\theta_m(\eta) - \theta_d(\eta))d\eta,$$

- Closed loop:

$$\Theta_m(s) = \frac{k_p s + k_I}{Js^3 + (B + k_d)s^2 + k_p s + k_I}\Theta_d(s) - \frac{s}{Js^3 + (B + k_d)s^2 + k_p s + k_I}D(s).$$

- If system is stable, $\theta_m(t) \to \theta_d(t)$, even when non-zero constant disturbance present
- Stability requires $k_I < \frac{k_p(B+k_d)}{J}$
  - We can use lower gains to track set-point θ_d
  - However, may need to know J, B to avoid instability
  - Also, integrators interact poorly with actuator saturation (wind-up problem)

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  - Eg:

$$\text{Model: } (Js^2 + B)\Theta_m(s) = U(s) - D(s)$$
$$\Longrightarrow b(s) = (Js^2 + B)$$
$$\Longrightarrow u_{FF}(t) = J\ddot{\theta}_d(t) + B\dot{\theta}_d(t)$$