ME 599/699 Robot Modeling & Control
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Gravity-Compensated PD Control

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- When

$$q_d(t) \equiv q_d,$$

  a constant, we get set-point regulation or goal-reaching task

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  - Lyapunov-based analysis and design

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Closed-loop:

$$M(q(t))\ddot{q}(t) + C(q(t), \dot{q}(t))\dot{q}(t) = K_P(q_d - q(t)) - K_D\dot{q}(t)$$

$$\implies \ddot{q}(t) = M^{-1}(q(t))\left(-C(q(t), \dot{q}(t))\dot{q}(t) + K_P(q_d - q(t)) - K_D\dot{q}(t)\right)$$

dropping t , $\ddot{q} = M^{-1}(q)\left(-C(q, \dot{q})\dot{q} + K_P(q_d - q) - K_D\dot{q}\right)$

Analysis

$$\ddot{q} = M^{-1}(q)\left(-C(q,\dot{q})\dot{q} + K_P(q_d - q) - K_D(\dot{q})\right)$$

Equilibrium occurs when $\dot{q} = \ddot{q} = 0 \implies q_{eq} = q_d$.

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Solution: Lyapunov methods

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Is this a proper candidate Lyapunov function?

- Need $K_P > 0$, $M(q) > 0$ (positive definite)

$M(q) > 0$ is true for any valid Euler-Lagrangian mechanical system!

Directional Derivative of Lyapunov Function

$$V(x) = V(q, \dot{q}) = \frac{1}{2}\dot{q}^T M(q)\dot{q} + \frac{1}{2}(q - q_d)^T K_P (q - q_d)$$

How does $V(x)$ change along solutions $\bar{x}(t)$?

$$\dot{V}(t) = \frac{\partial V}{\partial x}\dot{x}$$

$$= \dot{q}^T M(q)\ddot{q} + \frac{1}{2}\dot{q}^T \dot{M}(q)\dot{q} + (q - q_d)^T K_P \dot{q}$$

Next: substitute for $\ddot{q}$

$$\ddot{q} = M^{-1}(q)\left(-C(q,\dot{q})\dot{q} + K_P(q_d - q) - K_D\dot{q}\right)$$

$$\dot{V}(t) = \dot{q}^T M(q)\ddot{q} + \frac{1}{2}\dot{q}^T \dot{M}(q)\dot{q} + (q - q_d)^T K_P \dot{q} \quad (1)$$

$$= \dot{q}^T M(q)\left(M^{-1}(q)\left(-C(q,\dot{q})\dot{q} + K_P(q_d - q) - K_D\dot{q}\right)\right) \quad (2)$$
$$+ \frac{1}{2}\dot{q}^T \dot{M}(q)\dot{q} + (q - q_d)^T K_P \dot{q}$$

The mass-matrix terms cancel, so does the term involving K_P.
Exercise: confirm that you get from the equation above to:

$$\dot{V}(t) = \frac{1}{2}\dot{q}^T\left(\dot{M}(q) - 2C(q,\dot{q})\right)\dot{q} - \dot{q}^T K_D \dot{q}$$

Skew Symmetry Property

$$\dot{V}(t) = -\dot{q}^T K_D \dot{q},$$

because for any EL-system, $\dot{M}(q) - 2C(q, \dot{q})$ is a skew-symmetric matrix!
(See Section 5.2.1 in 07_Manipulator_Kinematics_Dynamics.pdf)

So, if $\dot{q} \neq 0$, then $\dot{V} < 0$.
To apply Lyapunov's conclusions, we actually want $q \to q_d$ is that when $q \neq q_d, \dot{q} \neq 0$, THEN $\dot{V} < 0$.

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A solution comes through La Salle's invariance principle (Hello again, ME 672).

Intuition: When its impossible for $\dot{V}(t) = 0$ forever at any state where $V(q) \neq 0$, then $q \rightarrow q_d$.

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$$G(q_{eq}) = K_P(q_d - q_{eq}),$$

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Question: Will an integrator work to handle gravity, like in the case of independent joint control?