ME/AER 676 Robot Modeling & Control
Spring 2023

Planning As (Graph) Search

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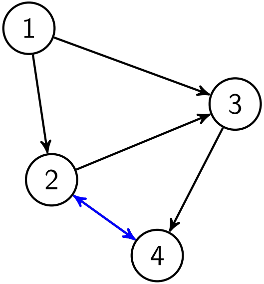
Graphs

A Graph $G = (V, E)$ Consists of two lists (sets):

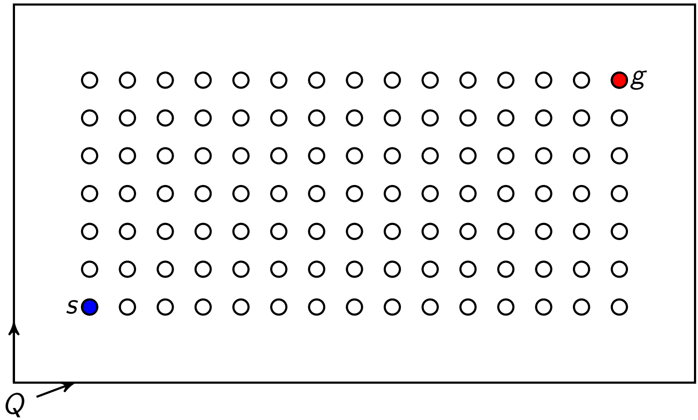
- Vertices V
  On the right, $V = \{1, 2, 3, 4\}$
- Edges $E \subseteq V \times V$
  On the right, $E = \{(1,2), (1,3), (2,3), (2,4), (3,4), (4,2)\}$



Graph Search

s = start, g = goal

Graph Search



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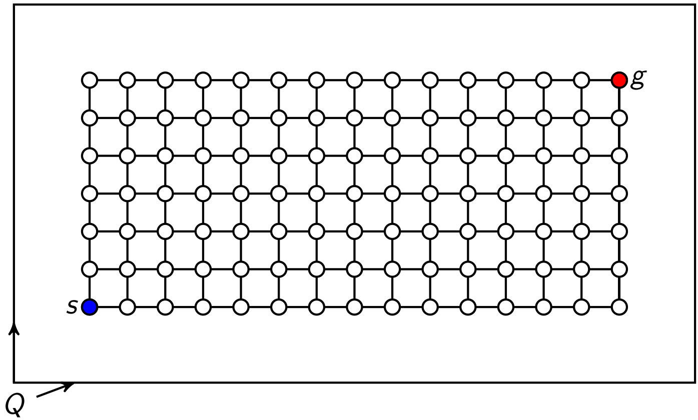
Define nodes

Graph Search



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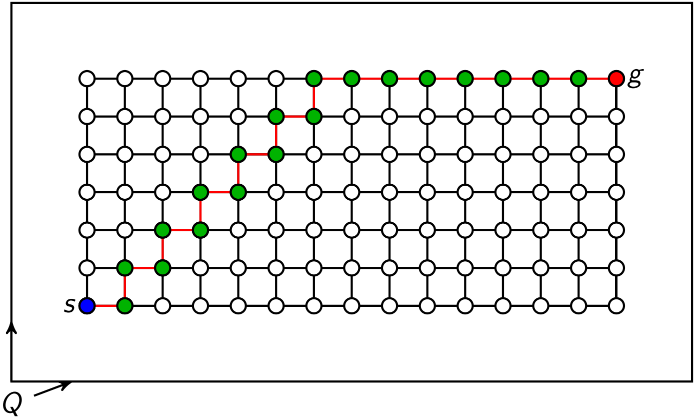
Graph Search



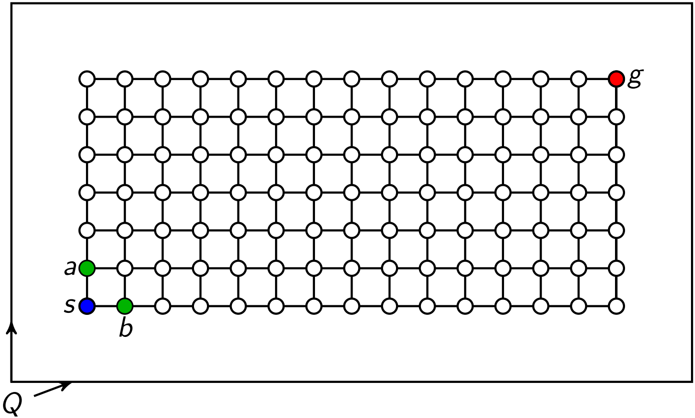
s = start, g = goal

Define nodes

Define edges

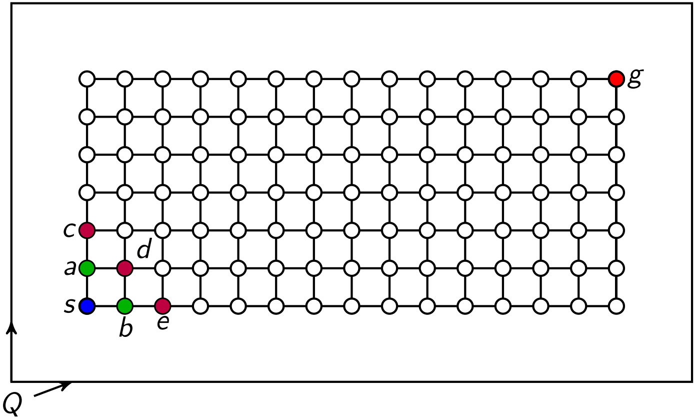
Find path(s)

Graph Search



s = start, g = goal
Define nodes
Define edges
Let's focus on the nodes close to s

Graph Search



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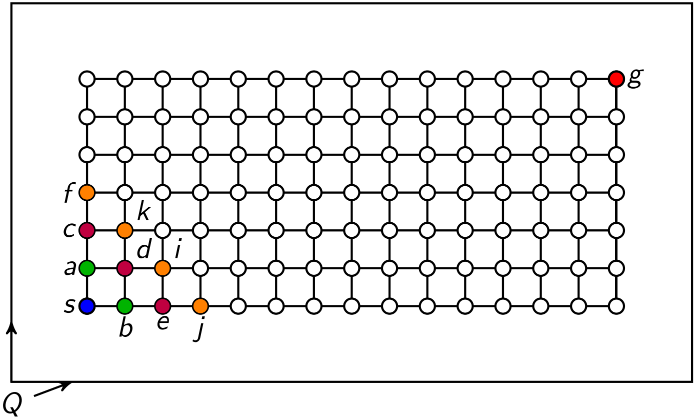

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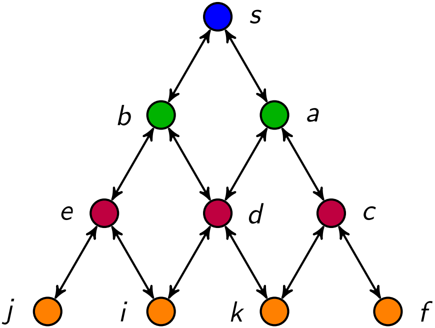
Graph As Seen From Start

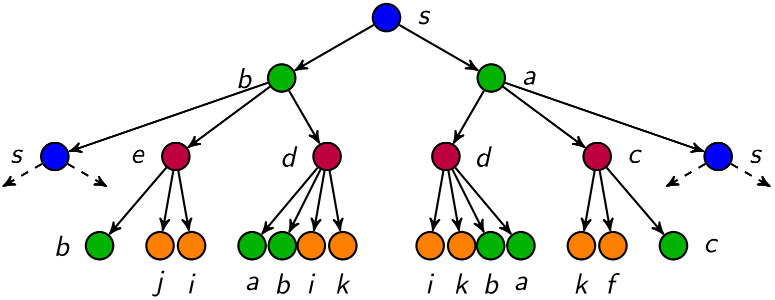


The undirected edges are equivalent to directed edges going forwards and back between two nodes.

We will use a search tree to traverse paths in this undirected graph.

Search Tree

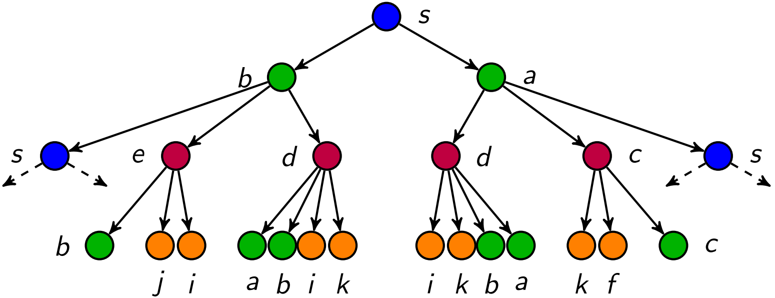
The root of the search tree is node s.
This tree depicts the possible paths starting from s.



Each 'layer' contains the nodes reached after taking a step from a node in a preceding layer.
Each layer has a depth, starting at 0 for s on top.
Node s appears at depth 0 and 2; b at 1 and 3, etc.

Search Tree

The root of the search tree is node s.
This tree depicts the possible paths starting from s.



The actual tree is much larger, with infinite depth.
For example, the tree recursively repeats itself whenever s appears at some depth.

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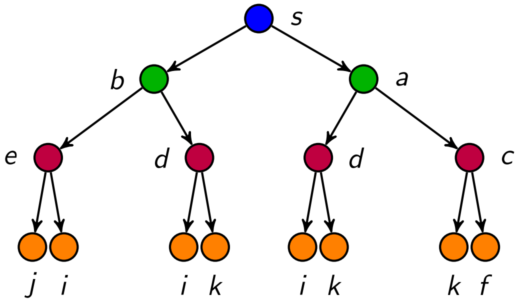
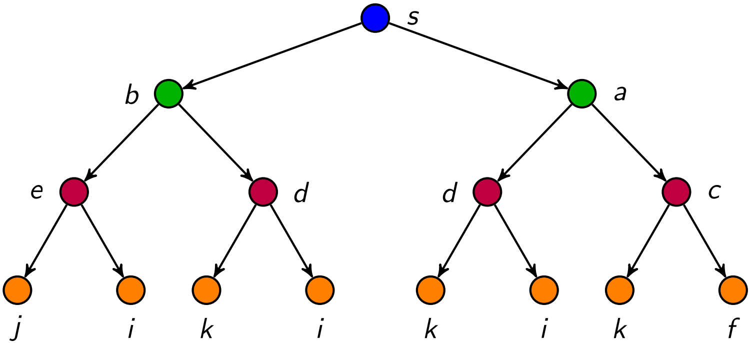


If the edges in the graph were directed, we would never return to nodes located at smaller depths, simplifying the tree.

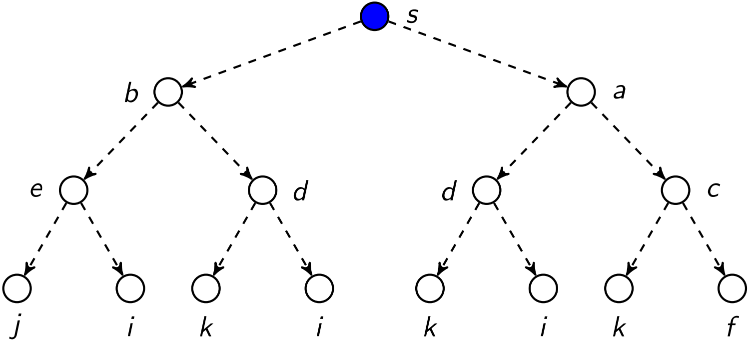
Search Tree



If a path exists from s to g, it is one of the possible paths in this search tree.

The key question is: how do we enumerate all the possible paths starting from s?

Search Algorithms



Search algorithms avoid representing all paths, or the entire tree, at one go.

Instead, the search tree is incrementally traversed.

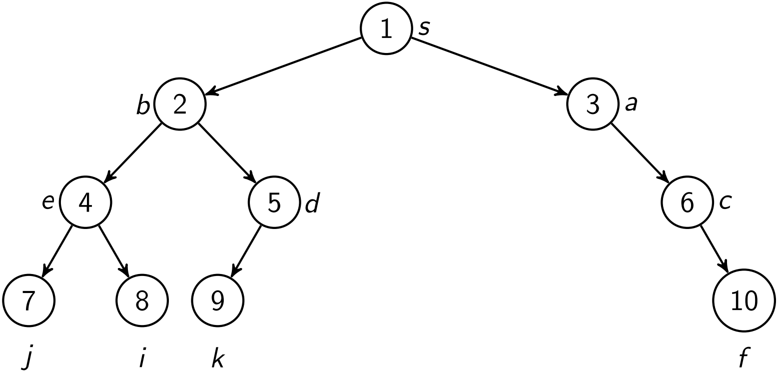
Start with root node, and no other nodes/edges.

Search Algorithms

Search algorithms differ in the order in which paths are evaluated, which boils down to the order in which unvisited successor states are checked for being the goal.

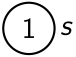
- Uninformed Search: Only know the successors at each node
  - Breadth First Search: Adds successors to end of list (Queue: first in first out)
  - Depth First Search: Adds successors to start of list (Stack: first in last out)
- Informed Search: Use an estimate of potential value of a node to order unvisited nodes (neither FIFO nor FILO).

Breadth-First Search



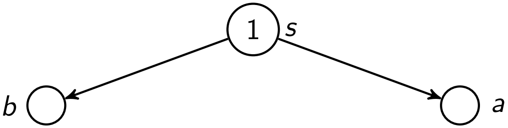
Order of visiting nodes under BFS. 'Row-by-row'

Breadth-First Search



What the search tree looks like as it is built

Breadth-First Search



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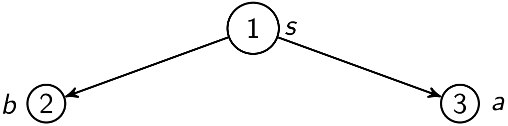

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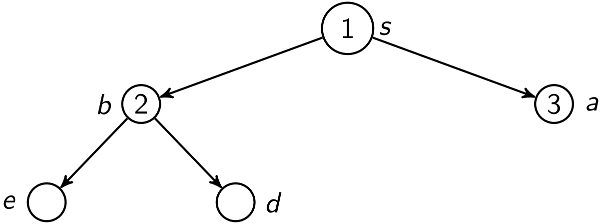

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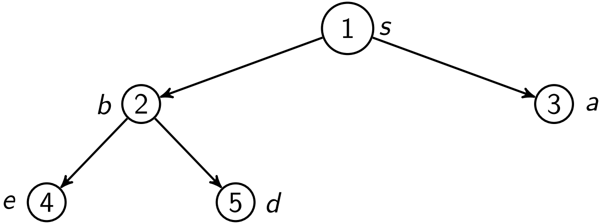

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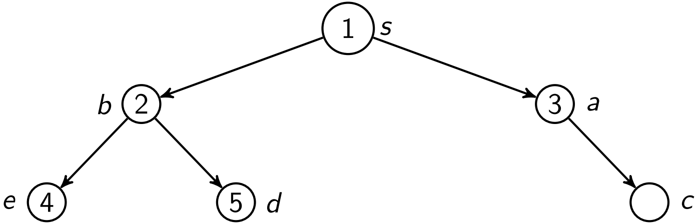

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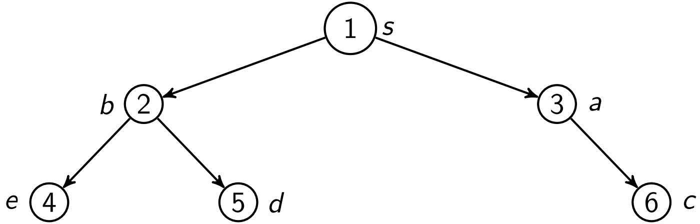

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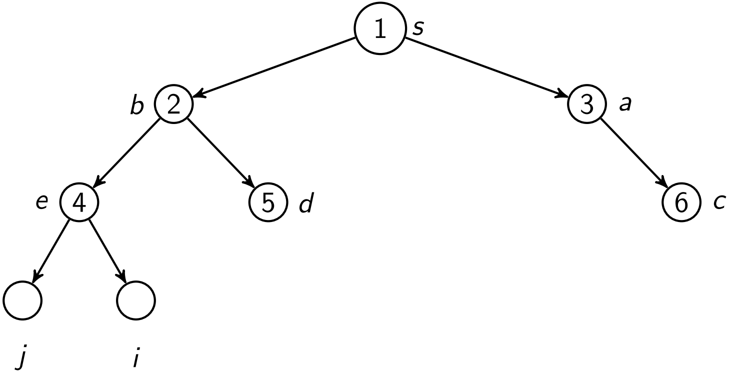

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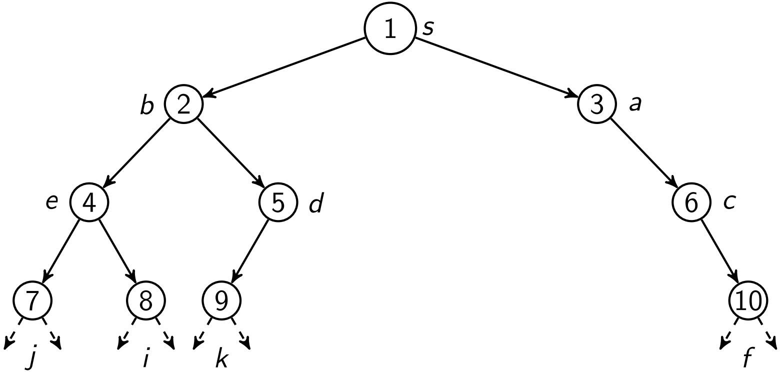

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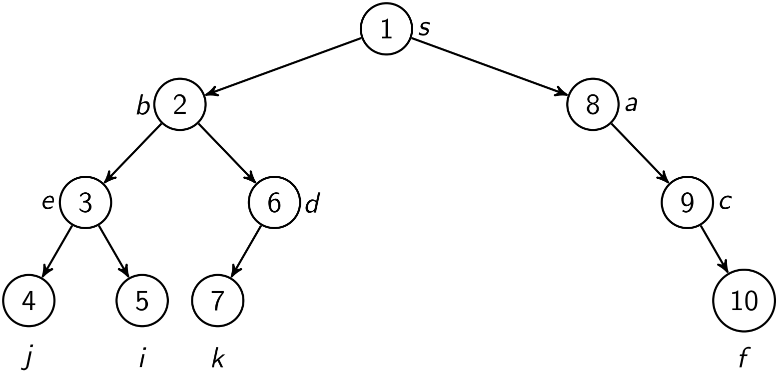

Order of visiting nodes under DFS. ‘Column-by-column’

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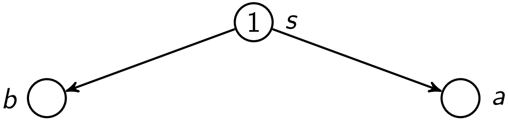

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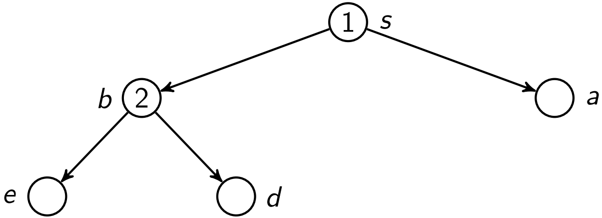

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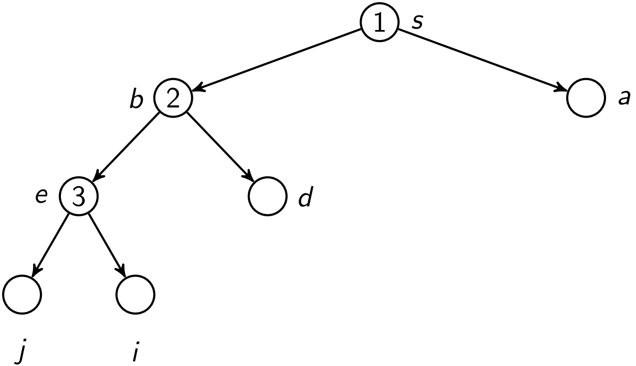

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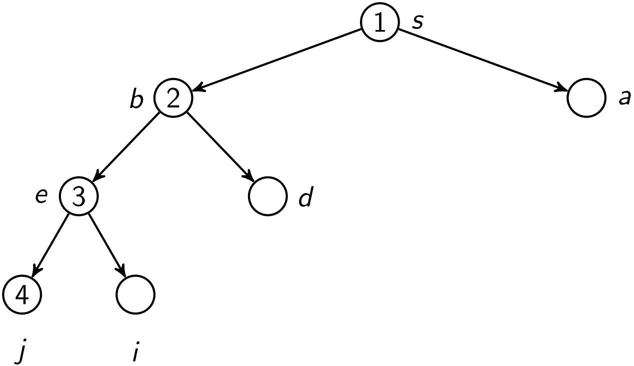

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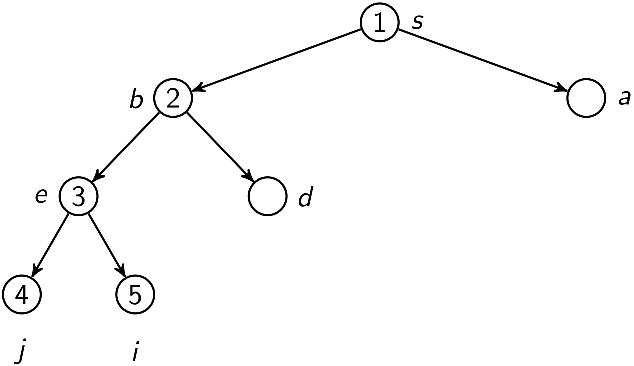

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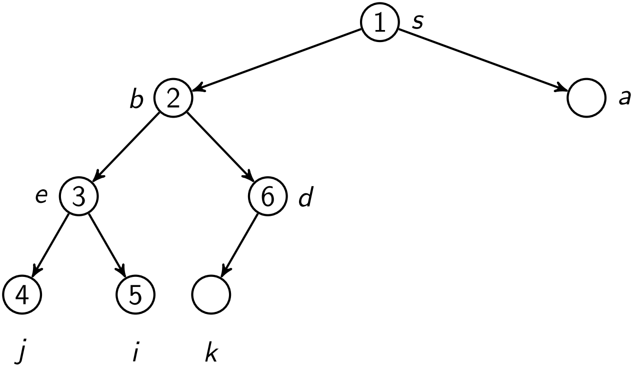

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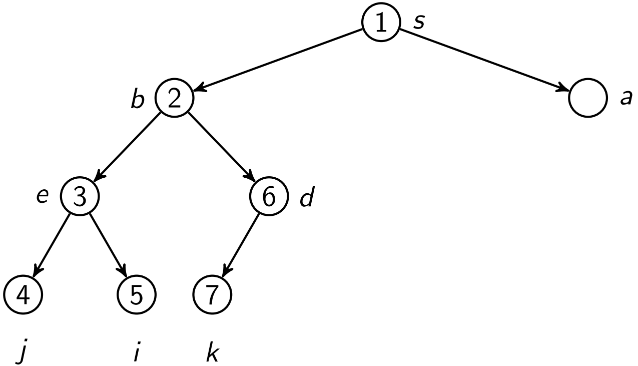

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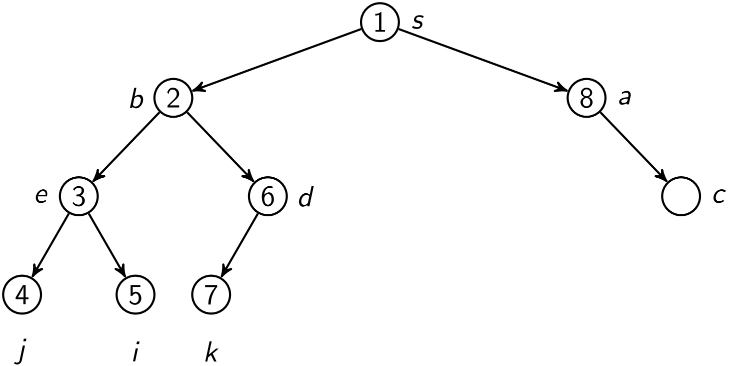

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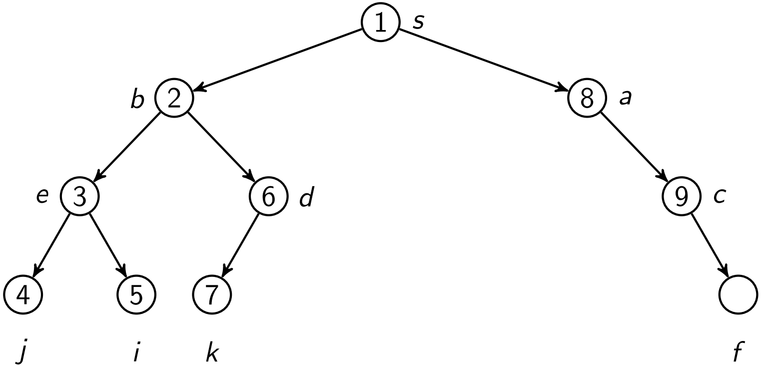

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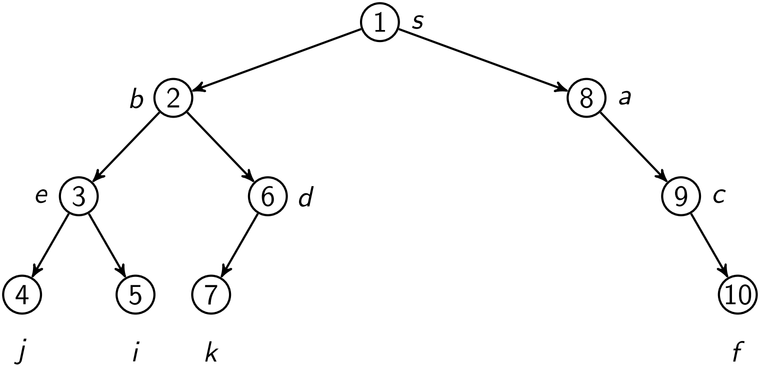

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- Some estimate of how likely a node will lead to the goal would be a useful way to break ties
- This idea leads to informed search, where the estimate for each node is provided by a *heuristic* function

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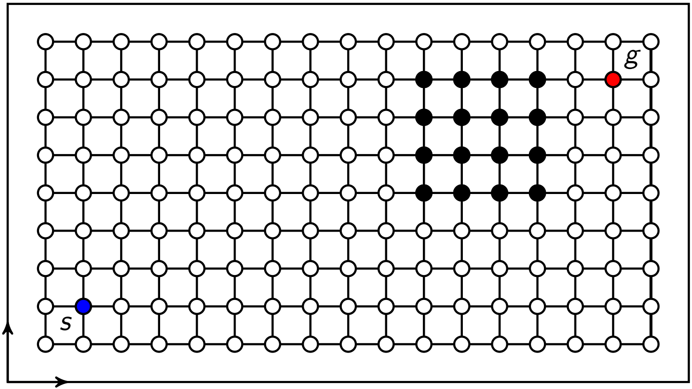
2. Djikstra's algorithm: order using $c(n)$
3. A*: order using $c(n) + h(n)$. Guaranteed performance when

$$h(n) \leq h^\star(n).$$

Motion Planning Discrete Space



Black nodes are 'obstacle' nodes

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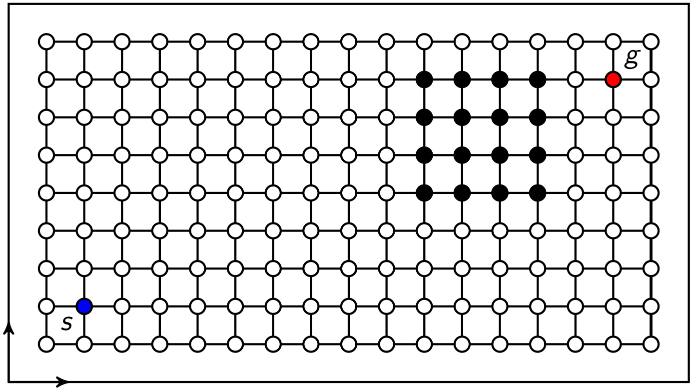

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While we ‘see’ the graph all at once, our algorithms deal with this graph using a search tree