

# ME/AER 676 Robot Modeling & Control

## Spring 2023

### Motion Planning Overview

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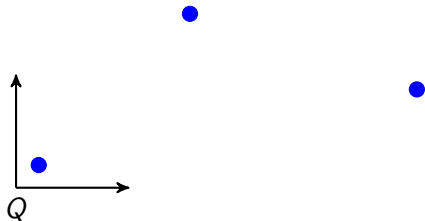
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Two types:

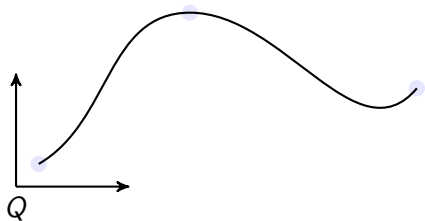
- ▶ Path Plan
- ▶ Trajectory Plan

# Path vs Trajectory

Discrete Path:  $\mathbb{N} \rightarrow Q \subset \mathbb{R}^n$



Continuous Path:  $I \rightarrow Q \subset \mathbb{R}^n$   
 $I$  is an interval,  $I \subseteq \mathbb{R}$



Trajectory  $I$  is a time interval

# Paths and Trajectories

## Definition (Discrete Path)

A path is a sequence of configurations.

## Definition (Continuous Path)

A path in  $\mathbb{R}^n$  is a continuous function  $\gamma$  from the unit interval  $I = [0, 1]$  to  $\mathbb{R}^n$ .

## Definition (Trajectory)

A trajectory  $q(t)$  in  $\mathbb{R}^n$  is a continuous function  $q$  from the an interval of time  $[t_0, t_f]$  to  $\mathbb{R}^n$ .

Some robots will execute motion described either as a path (waypoints) or a trajectory.

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  - ▶ Splines / Bezier curves
  - ▶ Polynomial trajectories

# Graphs

A Graph  $G = (V, E)$  Consists of two lists (sets):

- ▶ Vertices  $V$

On the right,  $V = \{1, 2, 3, 4\}$

- ▶ Edges  $E \subseteq V \times V$

On the right,  $E =$

$\{(1, 2), (1, 3), (2, 3), (2, 4), (3, 4), (4, 2)\}$

