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This Slide Deck

- Permanent Magnet DC Motors
  - Most small / hobby DC motors
  - Quadrotors, Mobile Robots, Robot Arms
- Block Diagram
- Servo Motor Control

Permanent Magnet Direct Current Motor

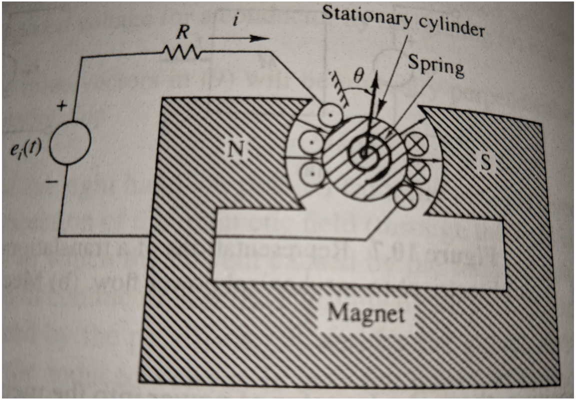

Figure: A PMDC (called a Galvanometer in Text) in Figure 10.8

Permanent Magnet Direct Current Motor

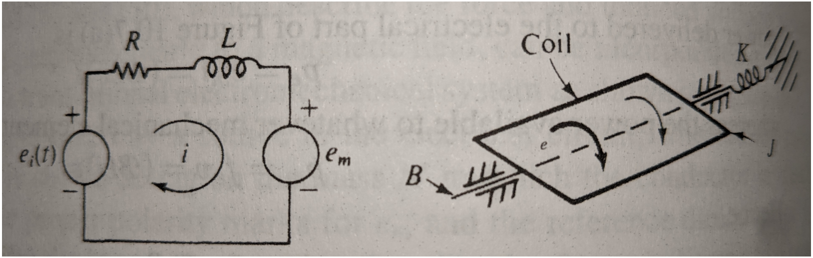

Figure: A lumped eletrical/mechanical model of a PMDC Motor

$$L\frac{\mathrm{d}i}{\mathrm{d}t} + Ri + e_{\text{motor}}(t) = e_i(t) \tag{1}$$

$$J\dot{\omega} + B\omega = \tau_{\text{electric}} + \tau_L(t), \text{ where} \tag{2}$$

$$\omega = \frac{\mathrm{d}\theta}{\mathrm{d}t}, \quad e_{\text{motor}} = \alpha\omega, \quad \tau_{\text{electric}} = \alpha i(t), \text{ and}$$

Also, instead of a spring torque $K\theta$ on the galvanometer, we use a generic load torque $\tau_L(t)$

PMDC Model

$$L\frac{\mathrm{d}i}{\mathrm{d}t} + Ri + \alpha\omega = e_i(t) \tag{3}$$

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We can obtain the transfer functions $G_1(s)$ and $G_2(s)$ from the inputs applied voltage $e_i(t)$ and load torque $\tau_L(t)$ to the output ω (rotation speed):

$$\begin{aligned}\hat{\omega}(s) &= {\color{red}G_1(s)}\hat{e}_i(s) + {\color{blue}G_2(s)}\hat{\tau}_L(s) \\ &= \frac{\alpha}{JLs^2 + (JR + BL)s + RB + \alpha^2}\hat{e}_i(s) \\ &\quad - \frac{Ls + R}{JLs^2 + (JR + BL)s + RB + \alpha^2}\hat{\tau}_L(s)\end{aligned} \tag{5}$$

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Note: 2^{nd} order model. Asymptotically stable for most physical values.

Analysis

$$\hat{\omega}(s) = G_1(s)\hat{e}_i(s) + G_2(s)\hat{\tau}_L(s) \qquad (6)$$

If we apply a constant load torque T_L and voltage E_i (step inputs), what would happen?

Analysis

$$\hat{\omega}(s) = G_1(s)\hat{e}_i(s) + G_2(s)\hat{\tau}_L(s) \tag{6}$$

If we apply a constant load torque T_L and voltage E_i (step inputs), what would happen? FVT!

$$\begin{aligned}
\hat{\omega}_{\text{step}}(\infty) &= \lim_{s\to 0}\left(s\left(G_1(s)\frac{E_i}{s} + G_2(s)\frac{T_L}{s}\right)\right) && (7)\\
&= G_1(0)E_i + G_2(0)T_L && (8)\\
&= \frac{\alpha E_i - RT_L}{RB + \alpha^2} && (9)\\
&= c_1 - c_2 T_L \qquad \text{(when } e_i(t),\ \tau_L(t) \text{ are constant)} && (10)
\end{aligned}$$

Steady-State Torque-Speed Curve

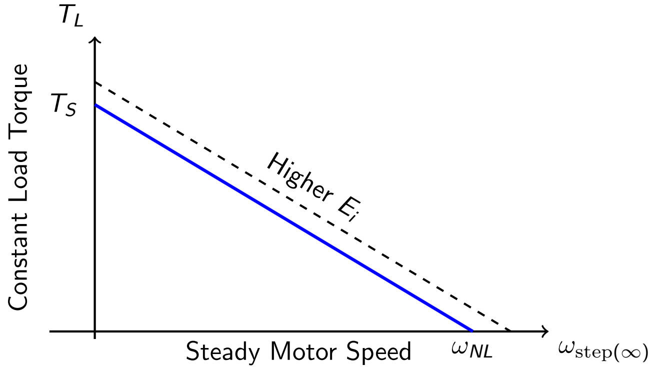


Figure: T_S: Stall Torque. ω_{NL}: No-Load Speed

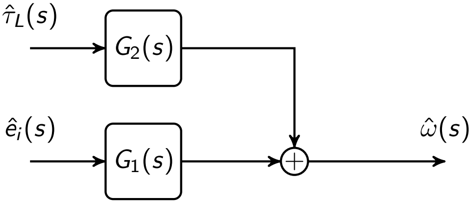
$$T_S = \frac{\alpha E_i}{R}, \quad \omega_{NL} = \frac{\alpha E_i}{RB + \alpha^2}$$

Recap

- Derived transfer functions (TF) from inputs τ_L and e_i to output $\omega = \dot{\theta}$
- Used TFs to predict steady-state speed under constant voltage and load torque.
- Used this prediction to draw a steady-state torque-speed curve.
- Looked at stall torque and no-load speed.

Speed Regulation

We can represent the derived transfer functions using a block diagram.



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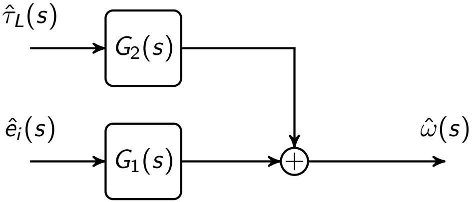

We can choose the input signal $\hat{e}_i(s)$, but not $\hat{\tau}_L(s)$.

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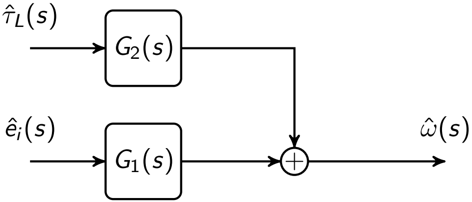

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Challenge: How do we ensure that $y(t) = \omega(t) \to \omega^*$, where ω^* is a desired steady speed?

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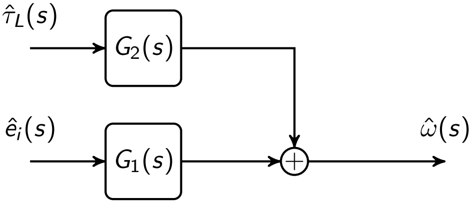

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Applications: Electrical power tools, CNC Machines, quadrotor/drones, radio-controlled planes/cars

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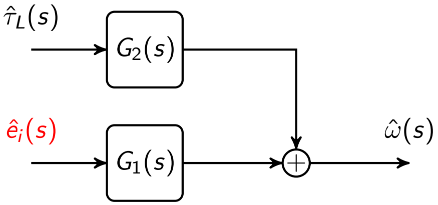
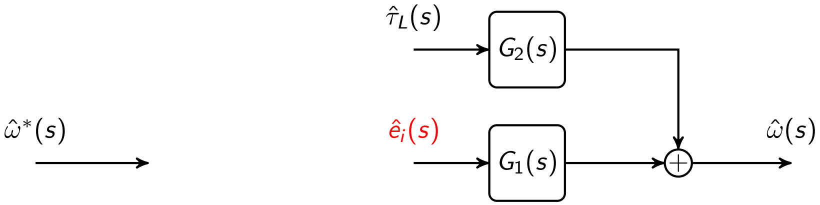


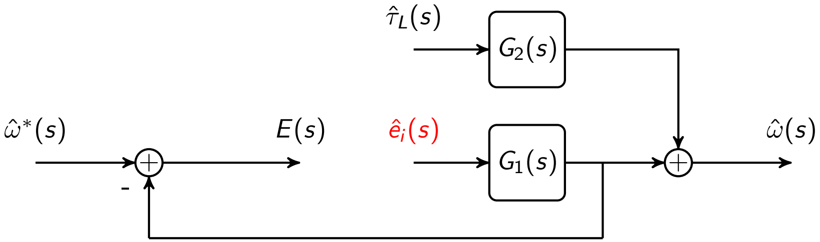
In hand-held power drills, we visually observe $\omega(t)$ and press a button to increase or decrease $\hat{e}_i(t)$

Speed Regulation



Feedback control is the automated version of this approach to regulating $\omega(t)$.

Speed Regulation



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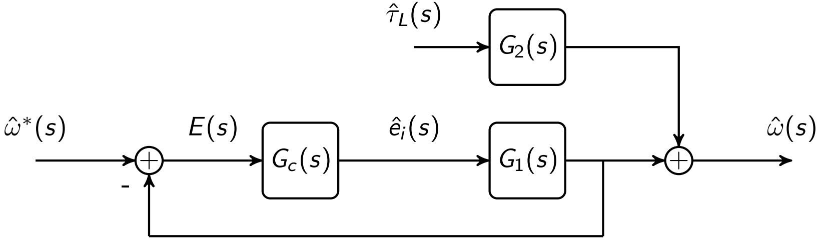
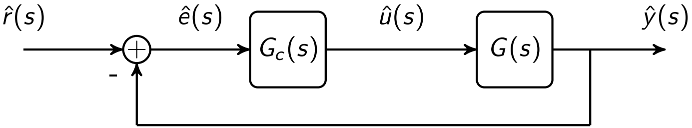


Feedback control is the automated version of this approach to regulating $\omega(t)$. $G_c(s)$ represents a **feedback control**.

Servo Loop



- $\hat{r}(s)$ is a reference signal
- $\hat{e}(s)$ is the error signal
- $\hat{u}(s)$ is the input or control signal
- $\hat{y}(s)$ is the output

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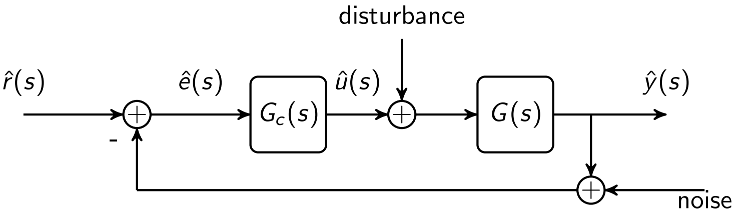

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