ME/AER 676 Robot Modeling & Control
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Models of Robotic Manipulators

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Tasks For Robots

- What kinds of tasks requires a physical robot?
  (Or, what can't your phone do?)
- How would you describe the motion you want to a robot?
- How would you, as a robot, convert that description into action?

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  We start with joint coordinates

Robot Configurations: Joint Variables

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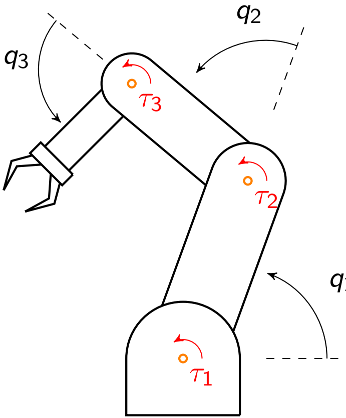

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Dynamics

The configuration q changes with time according to the second-order nonlinear ordinary differential equation

$$D(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \tau + \tau_{friction} + \tau_e, \tag{1}$$

where

- $D(q)$: inertia matrix (some formulations use $M(q)$)
- $C(q,\dot{q})$: Coriolis terms
- $G(q)$: the conservative forces (gradient of potential energy)
- τ is the force or torque vector generated by actuators at these joints
- $\tau_{friction}$ represents dissipative forces
- τ_e represents externally applied forces through contact with the environment

State Space Models

How do these dynamics compare to the usual state-space system?

$$\dot{x} = f(x, u)$$
$$y = h(x, u)$$

The configuration $q \in \mathbb{R}^n$ defines a state

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The output y above corresponds to the task space coordinate of the robot.
WARNING: Roboticists do not always use symbols x and y as above.

Goals

For this section of the course, we will

- Understand joint coordinates q
- Understand properties of the dynamics model (1)
- Understand the relationship $\chi = FK(q)$, where χ is the task coordinates (output) and FK is the forward kinematics map
- Derive controllers that make $q(t) \to q_d(t)$
- Derive controllers that make $\chi(t) \to \chi_d(t)$
- Derive controllers that enforce relationships between external forces τ_e and joint velocity $\dot{q}$ (Energy-based control)
- Convert this learning into simulations
- Understand challenges of perceiving the world

Goals

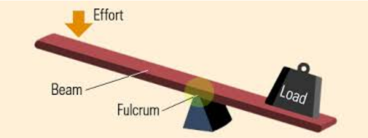
 

At the end of this part of the course, you should

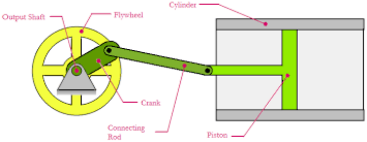
- understand how to derive equations and controllers in principle;
- know how to leverage software libraries to perform calculations involving robot models and controllers.

Robot Mechanisms

Mechanisms such as simple levers and crank-and-piston are widely used to transfer power/energy/force.



Simple lever



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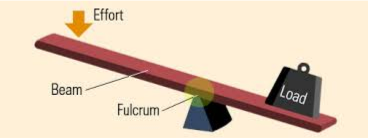

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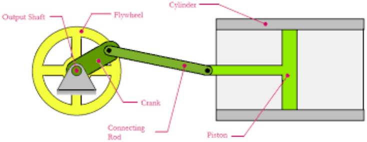


Crank and Piston

These mechanisms consist of (ideally) rigid bodies connected by joints that constrain relative motion.

Examples of Joints

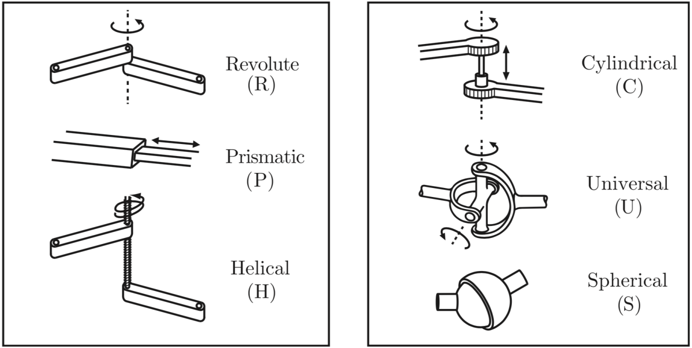


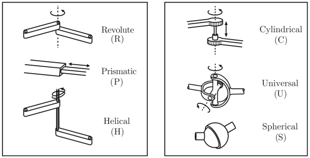
Figure 2.3: Typical robot joints.

source: Modern Robotics

Any joint can be abstractly replaced by multiple revolute (R) and/or prismatic (P) joints. Example: C= R+P

Joint Constraints

Joint type	dof f	Constraints c between two planar rigid bodies	Constraints c between two spatial rigid bodies
Revolute (R)	1	2	5
Prismatic (P)	1	2	5
Helical (H)	1	N/A	5
Cylindrical (C)	2	N/A	4
Universal (U)	2	N/A	4
Spherical (S)	3	N/A	3



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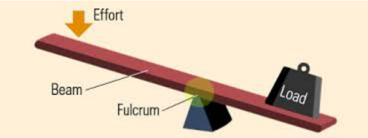
Exercise: Planar mechanisms

How many joints can you identify here? :



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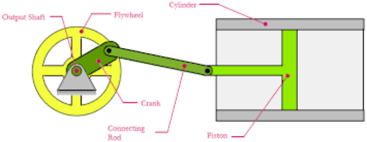

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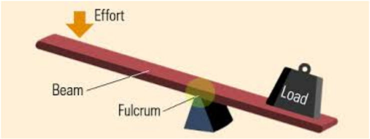

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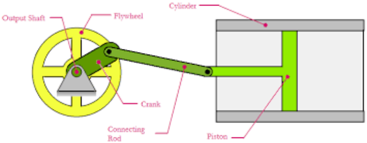
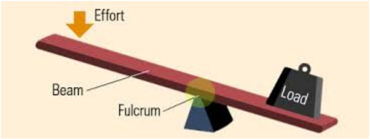


Crank and Piston

Lever: one revolute joint at the fulcrum.
Crank and Piston: Three revolute joints, one prismatic joint (formed by piston and cylinder)

Exercise: Planar mechanisms

What are the degrees of freedom of these mechanisms? :



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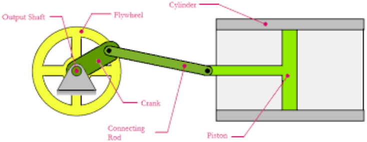

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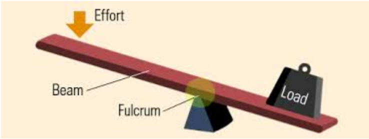

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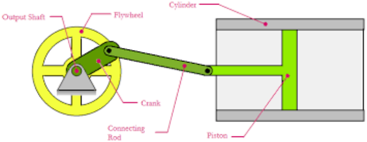

Crank and Piston

Lever: 1.
Why: the planar lever has 3DoF (two linear, one rotation). The revolute joint constrains 2DoF.

Crank and Piston: 1.
Why: Three moving links (green objects) = 9 DoF. The four joints restrict $4 \times 2 = 8$ DoF.

Kinematic Chains

We can combine links and joints in multiple ways:

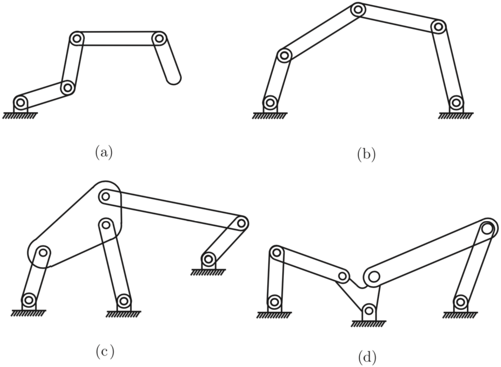

Figure 2.5: (a) k-link planar serial chain. (b) Five-bar planar linkage. (c) Stephenson six-bar linkage. (d) Watt six-bar linkage.

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Types of kinematic chains:

- Open / Closed
- Serial / Parallel

Idea: think of links forming a circuit including the non-moving fixed frame. Is it open/closed? Are links in series or parallel?

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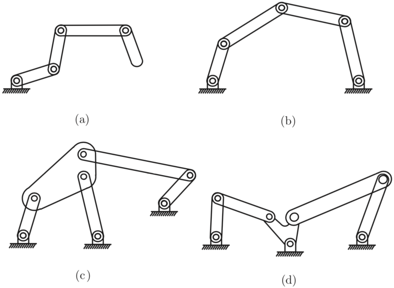
Idea: think of links forming a circuit including the non-moving fixed frame. Is it open/closed? Are links in series or parallel? Can you classify the mechanisms below?



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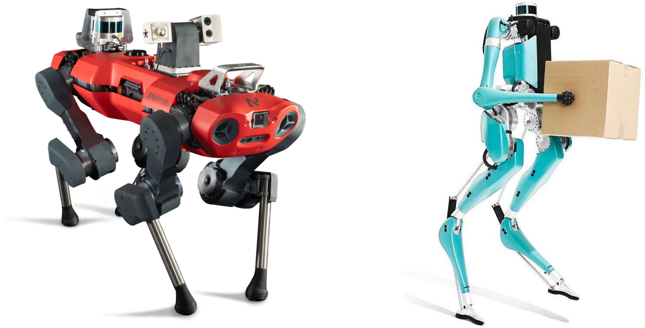

Figure: Kinematic Chains of Rigid Bodies

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In other words, we can change and measure the configuration q of a robot with revolute or prismatic joints in practice.

This fact is responsible for the development of robot modeling and control as it occurred, which happened before computer vision was as powerful as it is today.

Robot Configurations: Task Variables

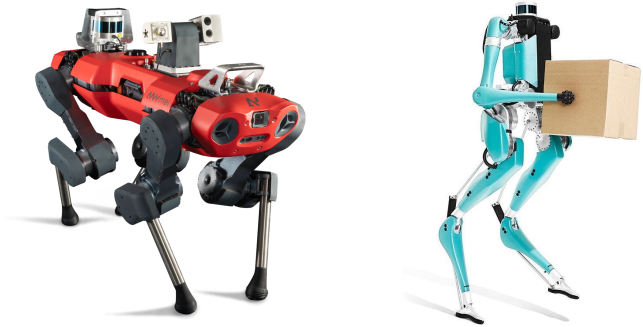

Figure: Powered Mechanisms

What do we want these powered mechanisms to do?

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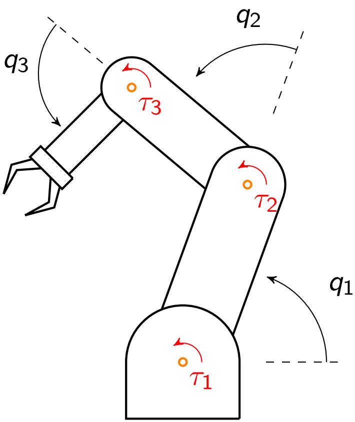

Joint Variables are 'internal' to the robot; Relevant for motion control.

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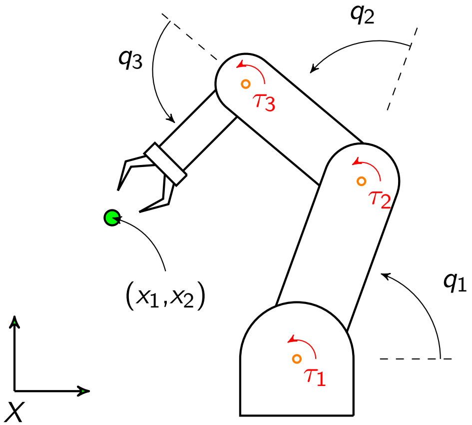
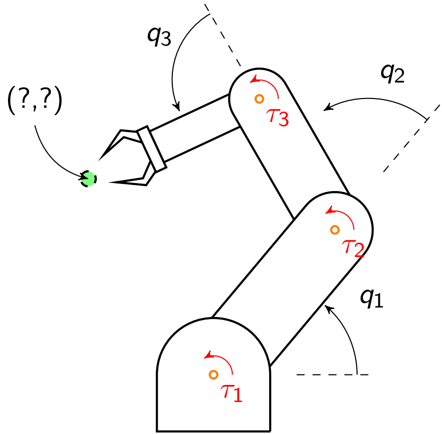


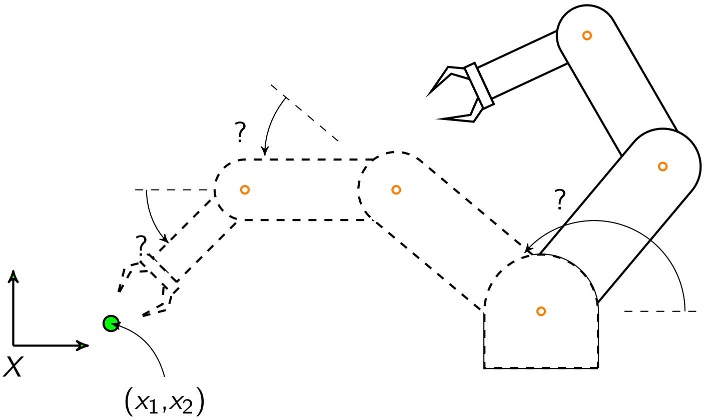
The task is to pick up an object at (x_1, x_2) defined in frame X. Therefore (x_1, x_2) forms the task coordinates χ.

Forward Kinematics



Given q, what is χ? Or, find f where $\chi = f(q)$.

Inverse Kinematics



Given χ, what is q? Or, find f^{-1} where $q = f^{-1}(\chi)$.

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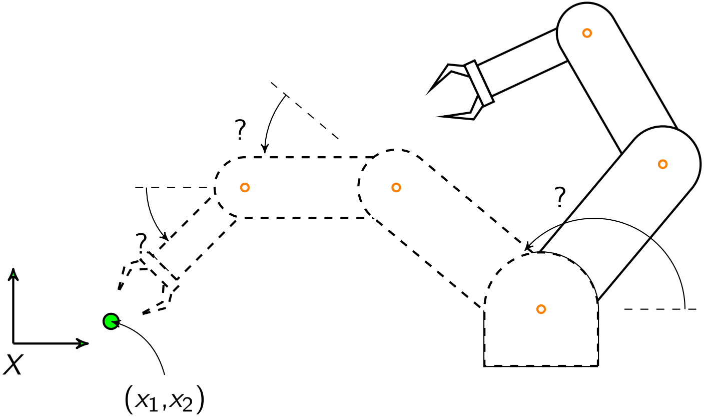

Switch between task trajectory $\chi(t)$ and joint trajectory $q(t)$.

$$D(q)\ddot{q} + C(q, \dot{q})\dot{q} = G(q) + \tau + \tau_{friction} + \tau_e, \qquad (2)$$