ME/AER 676 Robot Modeling & Control
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Drive Trains

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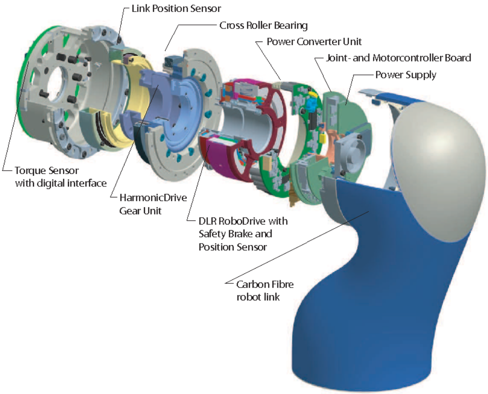
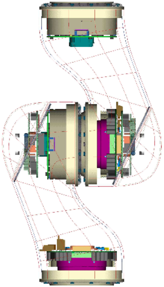

Link Position Sensor
Cross Roller Bearing
Power Converter Unit
Joint- and Motorcontroller Board
Power Supply
Torque Sensor
with digital interface
HarmonicDrive
Gear Unit
DLR RoboDrive with
Safety Brake and
Position Sensor
Carbon Fibre
robot link



Permanent Magnet DC motors

We consider permanent magnet DC motors. Other kinds include *AC* motors and brushless *DC* motors.

A model for the motor torque is $\tau = K_m i_a$, if the flux in the motor is constant. The current is generated by a voltage source, and has dynamics

$$L \frac{d}{dt} i_a + R i_a = V - V_b, \tag{1}$$

where L is the motor inductance, R is the winding resistance, V_b is the back EMF and is proportional to ω_m, the motor speed.

PMDC Model

The dynamics governing θ_m are

$$J_m\ddot{\theta}_m + B_m\dot{\theta}_m = \tau_m - \tau_l/r \tag{2}$$
$$= K_m i_a - \tau_l/r \tag{3}$$

We can rewrite (1) and (5) as

$$(Ls + R)I_a(s) = V(s) - K_b\ s\Theta_m(s), \tag{4}$$
$$(J_m s^2 + B_m s)\Theta_m(s) = K_m I_a(s) - \tau_l(s)/r \tag{5}$$

We combine these equations to obtain

$$s\left((Ls + R)(J_m s + B_m) + K_b K_m\right)\Theta_m(s) = K_m V(s) - \frac{(Ls + R)}{r}\tau_l(s). \tag{6}$$

Fast Electrical Dynamics

The electrical time constant L/R is much smaller than the mechanical time constant J_m/B_m. So, we can divide by R and set L/R to zero, obtaining.

$$s\left((J_m s + B_m) + \frac{K_b K_m}{R}\right)\Theta_m(s) = \frac{K_m}{R}V(s) - \frac{1}{r}\tau_l(s).$$

Setting $u \leftarrow K_m V/R$, and $B \leftarrow B_m + K_b K_m/R$, we obtain the motor equation as

$$J\ddot{\theta}_m + B\dot{\theta}_m = u(t) - \frac{1}{r}\tau_l. \tag{7}$$

Combined Link-Actuator Model

Let's combine the motor m with the link l

$$J_m\ddot{\theta}_m + B\dot{\theta}_m = u(t) - \frac{1}{r}\tau_l \quad (8)$$

$$J_l\ddot{\theta}_l + B_l\dot{\theta}_l = \tau_l \quad (9)$$

Due to the gears, $\dot{\theta}_m = r\dot{\theta}_l$. We eliminate θ_m and substitute $\theta_l \to q_k$ to obtain

$$\left(J_m r^2 + J_l\right)\ddot{q}_k + \left(Br^2 + B_l\right)\dot{q}_k = ru \quad (10)$$

Main takeaway: When gear ratio r is large, then link inertia becomes negligible compared to the motor's inertia.

Full Euler-Lagrangian Model

$$D(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \tau + \tau_{friction} + \tau_e,$$

becomes

$$\underbrace{M(q)}_{+\text{motor inertia}}\ddot{q} + C(q,\dot{q})\dot{q} + \underbrace{B\dot{q}}_{+\text{motor friction}} + G(q) = u + \tau_e,$$

where u denotes the input due to the voltage, whereas τ was the torque acting on the link joint.

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$$u_k = r_k \frac{K_{m_k}}{R_k} V_k,$$

where $\theta_{m_k} = r_k q_k$, and $M(q) = D(q) + J$, and J is diagonal with $r_k^2 J_{m_k}$ as k^{th} diagonal element.

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  - Brushless, QDD Motors, instrumental in 4-legged bots
- Huge boost in robotics due to better QDD motor supply chain